Trade, Productivity Improvements and Welfare: an Endogenous Market Structure Framework

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Abstract

In this paper, I investigate the welfare effects that developed countries experience after productivity improvements occur in their backward trading partners, using a two-country model featuring pro-competitive effects of trade and asymmetries in technology. I model the technology advantage of the leading country, assuming that the productivity distribution its firms draw from stochastically dominates that of the laggard country. Calibrated to match aggregate and firm level statistics of the US economy, the model predicts that the country with better technology has a higher productivity cutoff level, higher average productivity and higher welfare. Productivity improvements in the backward country generate selection and raise welfare everywhere, with both the selection effect and the positive welfare effect being stronger in the laggard country. Finally, trade liberalization is associated with more selection and higher welfare in both the leading and the laggard country.

Keywords: Asymmetric Countries, Productivity Improvements, Welfare, Endogenous Market Structure

1 Introduction

Recently, a new line of research revived a classic debate in international economics about the welfare effects developed countries experience after productivity improvements occur in their backward trading partners. This interest is driven by a series of recent developments in the world economy, such as a decline in trade costs and barriers, and an increase in market accessibility and in the spread of technology from the North to the South. Some of these studies rely on traditional trade models based on comparative advantage. Using a Ricardo-Mill framework, Samuelson (2004) simulates the effect on welfare in the US of a technology improvement in China, induced by imitation in the good

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in which the US previously had a comparative advantage. Results show that an expansion in China's labor productivity harms the US by causing a permanent loss in per capita real income. Jones and Ruffin (2008) show that under certain demand conditions and for a given range of relative country size, an advanced country benefits from an uncompensated technology transfer to a less advanced country. Paradoxically, this happens in the sector in which the advanced country has its greater comparative advantage. A number of empirical studies based on industry-level data have tested the predictions of such models. Bitzer et al (2008) test the predictions of Samuelson's paper for a group of OECD and developing countries, finding that knowledge spillovers from advanced to less advanced countries have a negative impact on output in the advanced countries. They also find that this negative effect is especially strong when knowledge transfer occurs towards China. Using a Ricardian-Heckscher-Ohlin model, di Giovanni et al (2011) find that the welfare effects generated by a productivity improvement in China substantially change across regions: most Asian countries (e.g. Malaysia and Taiwan) experience large positive welfare effects, whereas for many Latin American countries (e.g. Honduras and El Salvador) the welfare effects are negative. Finally, Levchenko and Zhang (2011) find that changes in developing countries' comparative advantage have virtually no impact on OECD countries, with a median welfare impact of zero and a very narrow range of variation across countries (from -0.2% to +0.6%). Other contributions have emphasized the importance of specific dimensions that have been neglected in traditional trade models. In a recent paper, Demidova (2008) highlights the role of "technological potential" in trade, which consists in the distribution of productivities that firms in each country draw from and the impact of this on competitiveness in the market. Demidova shows that if countries have different productivity distributions in terms of hazard rate stochastic dominance (HRSD) and in absence of specialization, then productivity improvements in one country raise welfare there but reduce that of its trading partner. Using a model featuring inter-industry trade, intra-industry trade and firm heterogeneity, Hsieh and Ossa (2011) capture productivity growth externalities through changes in the gains from comparative advantage (terms-of-trade effects), and through changes in the gains from increased variety and increased industry productivity (home market effects). They estimate China's productivity growth at the industry level, and quantify the welfare effects for China and the rest of the world generated by an increase in China's productivity. They find that only 3% of the worldwide gains of China's productivity growth spills over to other countries. Their analysis also reveals that some countries experience positive welfare effects (e.g. Japan and United States), whereas others experience negative effects (e.g. Russia and France).

This paper fits into this new line of research, proposing a novel framework to answer this classical question. I use an industry model with heterogeneous firms based on that of Impullitti and Licandro (2010), where trade liberalization has pro-competitive effects. Impullitti and Licandro use an oligopolistic framework to obtain an endogenous market structure, following a class of static trade models where the response of the market structure is driven by the strategic interaction of firms (Brander and Krugman, 1983; Venables, 1985; Neary, 2002, 2009). This is a more general framework than that proposed by Melitz and Ottaviano (2008), where the endogenous market structure is obtained by combining a particular form of preferences with a monopolistic competition framework. In Impullitti and Licandro, when an economy moves from autarky to trade, the number of firms operating in each local market doubles, thereby increasing product market competition. In this setting, trade liberalization generates two effects: a reduction in markups with a decrease in the inefficiency of oligopolistic markets, followed by an increase in firm's incentive to innovate (*direct competition effect*), and a selection effect (*selection effect of competition*), since the least productive firms exit the market as result of a greater product market competition. In my paper, there are two main differences with respect to Impullitti and Licandro (2010). First, I use a static version of their model, without innovation and growth. Second, I consider a model with only two countries that differ in their "technological potential". I am using the same definition of "technological potential" as introduced by Demidova, i.e. the productivity distribution firms in each country draw from. In particular, I assume that one of the two countries has a higher technological potential (better productivity distribution in terms of HRSD) than the other. This implies that firms in the country with higher technological potential have a better chance of drawing a higher level of productivity than firms in the other country, for any given level of productivity. Using a static model with endogenous market structure and only two countries having different technology allows me to analyse in a tractable framework the welfare effects of productivity improvements in backward countries, where new interesting mechanisms are at work. Although I use the same definition of "technological potential" as introduced by Demidova, my model is substantially different. Demidova uses a monopolistic competition model with heterogeneous firms based on Melitz (2003) to identify a technological potential effect. In this paper, I explore instead the properties of a new model where trade liberalization has also pro-competitive effects, and where welfare is affected through different channels.

The paper starts with the description of the closed economy case. I show that in equilibrium a better technology leads to a higher productivity cutoff level and higher average productivity. By means of a simple calibration based on firm-level and aggregate statistics of the US economy, I also show that welfare is lower in the backward country and decreasing in the technology gap. The second step consists in deriving the open economy equilibrium in a world with two countries having different technologies. I assume that one of the two countries (home) has a higher technological potential (better productivity distribution) than the other (foreign). The two countries engage in costly trade (iceberg type) with no entry costs in the export market. By means of a numerical simulation, I find that the advanced country has a higher productivity cutoff level, higher average productivity and higher welfare. Productivity improvements in the backward country generate a selection effect and raise welfare everywhere. However, both the selection effect and the positive welfare effect are stronger in the laggard country than in the leading country. Finally, I simulate trade liberalization scenarios for a given productivity gap, finding that a reduction in trade costs leads to more selection and increases consumers' welfare in both the leading and the laggard country.

2 The model

2.1 Preferences

In the economy there is a continuum of consumers of measure one. Two types of goods are produced: a homogeneous good, taken as the numeraire, and a composite good produced with a continuum of varieties. Each consumer inelastically supplies one unit of labor and has the following utility function:

$$U = lnX + \beta lnY \tag{1}$$

Y is the homogeneous good produced under constant returns to scale: a unit of labor can be transformed one-to-one into the homogeneous good.

The differentiated good X is produced with a continuum of varieties of endogenous mass $M \in [0,1]$ according to

$$X = (\int_0^M x_j^{\alpha} dj)^{\frac{1}{\alpha}} \tag{2}$$

where $\frac{1}{(1-\alpha)}$is the elasticity of substitution across varieties, with $\alpha \in (0,1)$

Each variety is produced by n identical firms according to the following production technology (I omit index j and identify the variety with its productivity)

$$\tilde{z}^{-1}q + \lambda = y \tag{3}$$

where y represent inputs, $\lambda > 0$ is a fixed production cost and $\tilde{z}^{-1}q$ is the variable cost of the firm producing variety j with productivity $\tilde{z}$.

The representative household maximizes utility subject to its budget constraint. The corresponding first order conditions are:

$$Y = \beta E \tag{4}$$

$$p_j = \frac{E}{X^{\alpha}} x_j^{\alpha-1} \tag{5}$$

where p is the price of variety j and $E = \int_0^M p_j x_j dj$ is the total household expenditure on the composite good X. Log preferences imply the total expenditure on the homogenous good to be β times total spending in the composite good. Equation (5) corresponds to the inverse demand function of variety $j \in [0,1]$.

2.2 Production

Firms producing the same variety compete à la Cournot and maximize their profits, taking as given the production of their competitors $\hat{x}$. Firm m producing variety j solves the following problem:

$$\pi_{mj} = [(p_{mj} - \tilde{z}_{mj}^{-1})q_{mj} - \lambda] \tag{6}$$

st.

$$p_{mj} = \frac{E}{X^{\alpha}} x_{mj}^{\alpha-1}$$

$$x = \hat{x} + q$$

The corresponding first order condition is (let us suppress indexes m and j to simplify notation):

$$\tilde{z}^{-1} = \theta \frac{E}{X^{\alpha}} x^{\alpha-1} \tag{7}$$

where $\theta \equiv \frac{(n-1+\alpha)}{n}$ is the inverse of the markup that firms charged over the marginal cost. Firms producing the same variety are symmetric, implying $x = nq$. The demand for variable inputs is obtained substituting (7) into (2) (See the Appendix for the derivations):

$$\tilde{z}^{-1} q = \theta e \frac{z}{\bar{z}} \tag{8}$$

where

$$\bar{z} = \frac{1}{M} \int_{0}^{M} z dj \tag{9}$$

is the average productivity, $e = E/(nM)$ is expenditure per firm and $z = \tilde{z}^{\frac{\alpha}{1-\alpha}}$.

2.3 Equilibrium in a closed economy

Profits can be written as a linear function of the relative productivity:

$$\pi(\frac{z}{\bar{z}}) = (1-\theta)e\frac{z}{\bar{z}} - \lambda \tag{10}$$

Let z^{*}be the cutoff productivity making firms' profits equal to zero. Solving for e , I derive the exit condition (EC) which denotes a negative relation between e and z^{*}:

$$e = \lambda \frac{1}{1-\theta} \frac{\bar{z}}{z^{*}} \tag{11}$$

Let us assume that there is a mass of unit measure of potential varieties of which $M \in [0,1]$ are operative. Non operative varieties draw their productivities from a common distribution $\Gamma(z)$, which is assumed to be continuous in (z_{min}, ∞), with $0 \leq z_{min} \leq \infty$. Since any entering firm drawing a level of productivity below z^* will immediately exit, the equilibrium density distribution $\mu(z)$ is given by:

$$\mu(z) = \begin{cases} \frac{f(z)}{(1-\Gamma(z^*))} & if z \geq z^* \\ 0 & otherwise \end{cases}$$

The average productivity can now be written as a function of the productivity cutoff z^*:

$$\bar{z}(z^*) = \frac{1}{1-\Gamma(z^*)} \int_{z^*}^{\infty} z f(z) dz \tag{12}$$

Irrespective of their productivity, varieties exit the market at rate δ. In a stationary equilibrium, in any period, the mass of new successful entrants should exactly replace the firms who face the bad shock and exit, hence:

$$(1-M)(1-\Gamma(z^*)) = \delta M \tag{13}$$

From (13) the mass of operative varieties is:

$$M(z^*) = \frac{1-\Gamma(z^*)}{1+\delta-\Gamma(z^*)} \tag{14}$$

The market clearing condition (MC) for the homogeneous good is:

$$n \int_0^M y_j dj + Y = n \int_0^M (\tilde{z}^{-1} q + \lambda) dj + \beta E = 1 \tag{15}$$

After changing the integration domain from sector $j \in [0,1]$ to productivities $z \in [z*, \infty]$, the market clearing condition becomes:

$$\int_{z^*}^{\infty} [\theta e \frac{z}{\bar{z}} + \lambda] \mu(z) dz + \beta e = \frac{1}{nM} \tag{16}$$

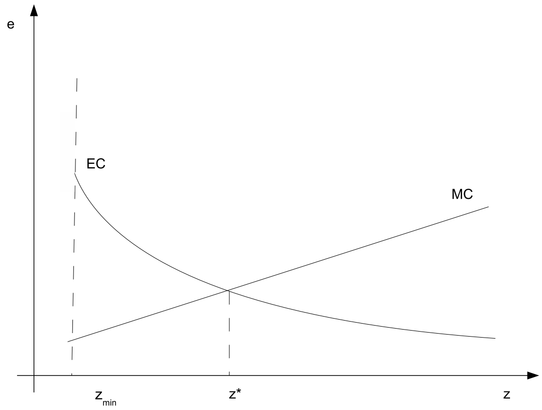
Since $\int_{z^*}^{\infty} \mu(z) dz = \int_{z^*}^{\infty} \frac{z}{\bar{z}} \mu(z) dz = 1$, after integrating over all sectors I obtain:

$$e = \frac{\frac{1}{nM(z^*)} - \lambda}{(\theta + \beta)} \tag{17}$$

Equation (17) denotes a positive relation between e and z^*. Assumption (1) guarantees the existence of a stationary equilibrium.

Assumption 1 *The entry distribution verifies, for all z,*

Figure 1: Equilibrium in closed economy



$$\frac{\bar{z}(z) - z}{\bar{z}(z)} \leq \frac{1 - \Gamma(z)}{zf(z)}$$

Assumption 1 makes $z^*/\bar{z}(z^*)$ increasing in z^* and therefore the (EC) curve decreasing in z^*.

Figure 1 provides a graphical representation of the equilibrium. An increase in the degree of competition (a reduction in the markup $1/\theta$), produced either by an increase in the substitutability parameter α or in the number of firms n, shifts both the (EC) and the (MC) curves to the right. Consequently z^* increases, therefore reducing the number of varieties $M(z^*)$, whereas the effect on e is ambiguous. In fact, depending on the relative strengths of the shift of the two curves, e can increase or decrease.

Using (1), (2), (4), and (7), I derive the indirect utility function as a measure of consumers' welfare

$$U = ln(\theta E(M\bar{z})^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E) \tag{18}$$

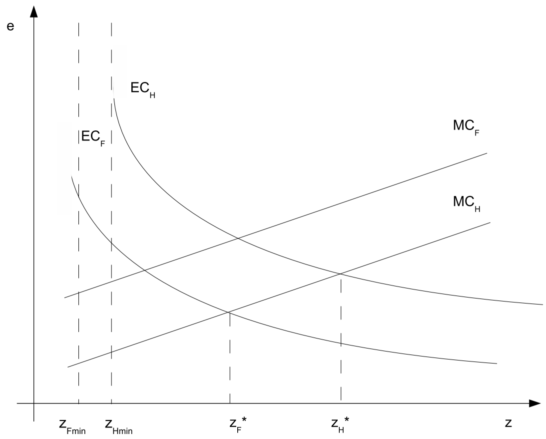
with $\alpha \in (0, 1)$

Welfare in each country depends on the inverse of the markup θ, on the number of active varieties M, on the average productivity $\bar{z}$ and on the total expenditure in the composite good E.

2.4 The effect of a better productivity distribution

In this section, I analyse the effect of a better productivity distribution on the equilibrium, without making any specific distributional assumption. Two

Figure 2: The effect of a better productivity distribution



closed economies are compared, assuming that one of them (home) has a better technology than the other (foreign).

Assumption 2: *The productivity distribution in the home country, $\Gamma_H(z)$, dominates the productivity distribution in the foreign country, $\Gamma_F(z)$, in terms of hazard rate stochastic dominance (HRSD), $\Gamma_H(.) \succ_{hr} \Gamma_F(.)$, if for any given level of productivity z*

$$\frac{f_H(z)}{1-\Gamma_H(z)} < \frac{f_F(z)}{1-\Gamma_F(z)}$$

Assumption 2 implies that for any given level of productivity z, firms in the home country have a better chance of drawing a level of productivity above this level than firms in the foreign country.

Proposition 1 *Under Assumption 2, for any given level of z, $EC_H > EC_F$ and $MC_H < MC_F$, thereby implying $z_H^* > z_F^*$.*

Proof See the Appendix.

Figure 2 provides a graphical representation of the equilibrium with the home country having a higher technological potential than the foreign country. In Section 4, I show through a numerical calibration that welfare is higher in the home country. I also show that welfare in the foreign country falls as the productivity gap increases (see Figure 3). Intuitively, firms in the home country are on average more productive and, in absence of trade, they face the same markup than firms in the foreign country (See equation 18).

3 Open Economy

Consider a world economy with two countries that have the same preferences and endowments as described in the previous section, but with different technologies. The home country has a superior technology, modeled in the form of a better productivity distribution its firms draw from in terms of HRSD. Trade costs are symmetric and of the standard iceberg type: $\tau > 1$ units shipped result in 1 unit arriving. As in the baseline model of Impullitti and Licandro (2010), there are no entry costs in the export market, so that all firms operate both in the domestic and the foreign market

3.1 Equilibrium characterization

Assumption 2 implies that firms producing the same variety, but located in different countries, have different marginal costs. As a consequence, there is no perfect overlap between the varieties produced in the two economies, as in Impullitti and Licandro (2010), because firms in sector j in country i might decide, given their draw, to exit, while their rivals in the other country might stay and produce in the same sector. Firms in sector j in country i face two possible scenarios: (i) they might be the only ones producing variety j, therefore serving both the domestic and the foreign market; (ii) they might produce variety j in competition with firms located in the other country, sharing with them both the domestic and the foreign market.

3.1.1 First scenario: varity j is produced only in the home (foreign) country

Let us consider the case in which variety j is produced only by firms in the home country. Let q_{HH} and q_{HF} be the quantities of variety j produced for the domestic and the foreign markets respectively. Each firm in the home country solves a problem which leads to the following first order conditions[1]

$$[(\alpha-1)\frac{q_{HH}}{x_H}+1]p_H=\frac{1}{\tilde{z}_H} \tag{19}$$

$$[(\alpha-1)\frac{q_{HF}}{x_F}+1]p_F=\frac{\tau}{\tilde{z}_H} \tag{20}$$

Variables x_H and x_F represent the total output offered and $p_H=\frac{E_H}{X_H^\alpha}x_H^{\alpha-1}$ and $p_F=\frac{E_F}{X_F^\alpha}x_F^{\alpha-1}$ are prices of variety j in the domestic and in the foreign market respectively. Firms in the home country entirely satisfy both the domestic and the foreign demand, implying $x_H=nq_{HH}$ and $x_F=nq_{HF}$. The resulting demand for variable inputs is

$$\frac{q_{HH}+\tau q_{HF}}{z_H}=\psi e\frac{z_H}{\bar{z_H}} \tag{21}$$

[1]The Appendix provides details of derivations.

where $\psi = \left[\frac{\alpha-1+n}{n}(1+\tau)\right]$ is the inverse of the average markup faced by a firm in the home country in both the domestic and the foreign market. Not surprisingly, the average markup corresponds to the markup faced by firms in the closed economy times $(1+\tau)$, which takes into account the transportation costs for the quantities sold into the foreign market. Profits of a firm in sector j in the home country are

$$\pi_H(\frac{z_H}{\bar{z}_H}) = (1+\tau-\psi)e\frac{z_H}{\bar{z}_H} - \lambda \tag{22}$$

The specular case is when variety j is produced only in the foreign country. In this case, profits of a firm in the foreign country producing variety j are

$$\pi_F(\frac{z_F}{\bar{z}_F}) = (1+\tau-\psi)e\frac{z_F}{\bar{z}_F} - \lambda \tag{23}$$

3.1.2 Second scenario: variety j is produced in both countries

The second scenario occurs when variety j is produced in both countries. In this case, firms in sector j in country i share the market with their rivals in the other country, and their profits are a function of the relative productivity gap γ_j in that sector. The relative productivity gap is defined as

$$\gamma_j = \frac{z_{jF}}{z_{jH}}$$

$0 < \gamma_j < \infty$, $\gamma_j = \tilde{\gamma_j}^{\frac{\alpha}{1-\alpha}}$

with cumulative distribution $G(\gamma)$ and a density $g(\gamma)$ (I keep on omitting index j, however each variety is now associated with two levels of productivity, one in the home country and one in the foreign country). Let q_{HH} and q_{HF} be the quantities of variety j produced for the domestic and for the foreign market by firms in the home country, and q_{FF}, and q_{FH} the quantities produced for the domestic and the foreign market by firms in the foreign market.

A firm in the home country producing variety j solves a problem which leads to the following first order conditions[2]

$$[(\alpha-1)\frac{q_{HH}}{x_H}+1]p_H = \frac{1}{\tilde{z}_H} \tag{24}$$

$$[(\alpha-1)\frac{q_{HF}}{x_F}+1]p_F = \frac{\tau}{\tilde{z}_H} \tag{25}$$

the corresponding first order conditions for a firm in the foreign country are

$$[(\alpha-1)\frac{q_{FF}}{x_F}+1]p_F = \frac{1}{\tilde{z}_F} \tag{26}$$

$$[(\alpha-1)\frac{q_{FH}}{x_H}+1]p_H = \frac{\tau}{\tilde{z}_F} \tag{27}$$

[2]The Appendix provides details of derivations.

Using $\tilde{z}_F = \tilde{\gamma}\tilde{z}_H$ and the first order conditions, the domestic and the foreign markups can be expressed in both countries as a function of the relative technology gap $\tilde{\gamma}$

$$\theta_{HH} = \frac{\alpha - 1 + 2n}{n}\left(\frac{\tilde{\gamma}}{\tilde{\gamma} + \tau}\right) \tag{28}$$

$$\theta_{HF} = \frac{\alpha - 1 + 2n}{n}\left(\frac{\tilde{\gamma}\tau}{1 + \tilde{\gamma}\tau}\right) \tag{29}$$

$$\theta_{FF} = \frac{\alpha - 1 + 2n}{n}\left(\frac{1}{1 + \tilde{\gamma}\tau}\right) \tag{30}$$

$$\theta_{FH} = \frac{\alpha - 1 + 2n}{n}\left(\frac{\tau}{\tilde{\gamma} + \tau}\right) \tag{31}$$

Since firms in the home country and firms in the foreign country have different marginal costs, they face different markups both in the domestic and in the foreign market. Furthermore, as in Impullitti and Licandro (2010), because of trade costs, firms located in the same country face different markups for the domestic and the foreign market. For any given level of productivity gap, the following inequalities hold: $\theta_{HH} < \theta_{HF}$ and $\theta_{FF} < \theta_{FH}$.

The demands for variable inputs in the home country and in the foreign country are

$$\frac{q_{HH} + \tau q_{HF}}{z_H} = \chi_H e \frac{z}{\bar{z}_H} \tag{32}$$

$$\frac{q_{FF} + \tau q_{FH}}{z_F} = \chi_F e \frac{z}{\bar{z}_F} \tag{33}$$

where:

$$\chi_H = \left\{ \tfrac{\alpha-1+2n}{n} \tfrac{1}{(\alpha-1)} \tfrac{\tilde{\gamma}}{(\tilde{\gamma}+\tau)} \left[\tfrac{\tilde{\gamma}(\alpha-1+n)-\tau n}{\tilde{\gamma}+\tau} + \tfrac{\tilde{\gamma}\tau(\alpha-1+n)-n}{1+\tau\tilde{\gamma}} \tau \right] \right\}$$
$$\chi_F = \left\{ \tfrac{\alpha-1+2n}{n} \tfrac{1}{(\alpha-1)} \tfrac{1}{(1+\tau\tilde{\gamma})} \left[\tfrac{\alpha-1+n-\tau\tilde{\gamma}n}{1+\tau\tilde{\gamma}} + \tfrac{\alpha-1+n-n\tilde{\gamma}}{\tilde{\gamma}+\tau} \tau \right] \right\}$$

Differently from Impullitti and Licandro (2010), χ_H and χ_F do not coincide with the inverse of the average markups as, due to asymmetry, total supply in country i, $x_i = n(q_{ii} + q_{li})$, does not correspond to total quantity produced there, $Q_i = n(q_{ii} + q_{il})$, with $i \neq l$.

The inverse of the average markup faced by firms in the home country and in the foreign country is a weighted sum of the domestic and of the foreign markup, where the weights are given by the relative quantities produced for the domestic and for the foreign market respectively[3]

[3] When $\tilde{\gamma} = 1$, that is countries are symmetric, $\theta_{\tau H}$ and $\theta_{\tau F}$ collapse into $\theta_\tau = \frac{2n-1+\alpha}{n(1+\tau)^2(1-\alpha)}[\tau^2(1-n-\alpha) + n(2\tau - 1) + (1-\alpha)]$, the average markup in Impullitti and Licandro (2010).

$$\theta_{\tau H} = \left[\frac{q_{HH}}{q_{HH}+q_{HF}}\frac{\alpha-1+2n}{n}(\frac{\tilde{\gamma}}{\tilde{\gamma}+\tau}) + \frac{\tau q_{HF}}{q_{HH}+q_{HF}}\frac{\alpha-1+2n}{n}(\frac{\tilde{\gamma}\tau}{1+\tilde{\gamma}\tau})\right] \quad (34)$$

$$\theta_{\tau F} = \left[\frac{q_{FF}}{q_{FF}+q_{FH}}\frac{\alpha-1+2n}{n}(\frac{1}{1+\tilde{\gamma}\tau}) + \frac{\tau q_{FH}}{q_{FF}+q_{FH}}\frac{\alpha-1+2n}{n}(\frac{\tau}{\tilde{\gamma}+\tau})\right] \quad (35)$$

Profits for a firm in the home country and for a firm in the foreign country are

$$\pi_H(\frac{z}{\bar{z}}) = (A-\chi_H)e\frac{z}{\bar{z}_H} - \lambda \quad (36)$$

$$\pi_F(\frac{z}{\bar{z}}) = (B-\chi_F)e\frac{z}{\bar{z}_F} - \lambda \quad (37)$$

where

$$A = \tfrac{1}{(\alpha-1)}\tfrac{1}{\tilde{\gamma}+\tau}\left[(1+\tau)\left((\alpha-1+n)\tilde{\gamma}-n\right)\right]$$
$$B = \tfrac{1}{(\alpha-1)}\tfrac{1}{1+\tau\tilde{\gamma}}\left[(1+\tau)\left((\alpha-1+n)-\tilde{\gamma}n\right)\right]$$

3.1.3 The equilibrium conditions

Firms in the home country and in the foreign country face these two events with different probabilities. Therefore, the profit function of a firm in sector j in country i is a weighted sum of the profits obtained in these two events, where the weights are given by the probability that sector j is active, $1-\Gamma_l(z_l^*)$, or not active, $\Gamma_l(z_l^*)$, in the other country with $l \neq i$. Profits when sector j is active in both countries are also weighted by the density function of the productivity gap $g(\gamma)$

$$\begin{aligned}\pi_H(\tfrac{z_H}{\bar{z}_H}) = &\left[e\tfrac{z_H}{\bar{z}_H}(1+\tau-\psi)-\lambda\right]\Gamma_F(z_F^*) + \\ + &\left[e\tfrac{z_H}{\bar{z}_H}\textstyle\int_o^\infty(A-\chi_H)g(\gamma)d\gamma-\lambda\right](1-\Gamma_F(z_F^*))\end{aligned} \quad (38)$$

$$\begin{aligned}\pi_F(\tfrac{z_F}{\bar{z}_F}) = &\left[e\tfrac{z_F}{\bar{z}_F}(1+\tau-\psi)-\lambda\right]\Gamma_H(z_H^*) + \\ + &\left[e\tfrac{z_F}{\bar{z}_F}\textstyle\int_o^\infty(B-\chi_F)g(\gamma)d\gamma-\lambda\right](1-\Gamma_H(z_H^*))\end{aligned} \quad (39)$$

Assumption 2 implies $\int_o^\infty(A-\chi_H)g(\gamma)d\gamma > \int_o^\infty(B-\chi_F)g(\gamma)d\gamma$ as for every z the home country has a better chance of drawing a higher level of productivity.

As in the closed economy, we derive the productivity cutoff in the two countries by the exit conditions which are

$$e_H = \frac{\lambda}{\left[(1+\tau-\psi)\right]\Gamma_F(z_F^*) + \left[\int_o^\infty (A-\chi_H)g(\gamma)d\gamma\right](1-\Gamma_F(z_F^*))} \frac{\bar{z}_H}{z_H^*} \tag{40}$$

$$e_F = \frac{\lambda}{\left[(1+\tau-\psi)\right]\Gamma_H(z_H^*) + \left[\int_o^\infty (B-\chi_F)g(\gamma)d\gamma\right](1-\Gamma_H(z_H^*))} \frac{\bar{z}_F}{z_F^*} \tag{41}$$

The market clearing conditions become

$$e_H = \frac{\frac{1}{M(z_H^*)} - \lambda}{\psi\Gamma_F(z_F^*) + \left[\int_o^\infty \chi_H g(\gamma)d\gamma\right](1-\Gamma_F(z_F^*)) + \beta} \tag{42}$$

$$e_F = \frac{\frac{1}{M(z_F^*)} - \lambda}{\psi\Gamma_H(z_H^*) + \left[\int_o^\infty \chi_F g(\gamma)d\gamma\right](1-\Gamma_H(z_H^*)) + \beta} \tag{43}$$

The equilibrium allocations for the home country and the foreign country are obtained by solving this system of four equations (40), (41), (42), and (43) and four unkowns: $z_H^*, z_F^*, e_H^*, e_F^*$. Since the equilibrium system is fairly complex, its properties are explored numerically in Section (4)

In the open economy, welfare for consumers in the home country and in the foreign country becomes

$$W_H = [ln(E_H\theta(M_H\bar{z}_H)^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_H)][\Gamma_F(z^*)] +$$

$$+[ln(E_H(\int_0^\infty \Phi_H g(\gamma)d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_H)][1-\Gamma_F(z^*)] \tag{44}$$

and

$$W_F = [ln(E_F\theta(M_F\bar{z}_F)^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_F)][\Gamma_H(z^*)] +]$$

$$+[ln(E_F(\int_0^\infty \Phi_F g(\gamma)d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_F)][1-\Gamma_H(z^*) \tag{45}$$

where

$\Phi_H = [\frac{\alpha-1+2n}{n}]\frac{\tilde{\gamma_i}}{\tilde{\gamma_i}+\tau}\frac{1}{1+\tilde{\gamma_i}}$
$\Phi_F = [\frac{\alpha-1+2n}{n}]\frac{\tilde{\gamma_i}}{1+\tilde{\gamma_i}\tau}\frac{1}{1+\tilde{\gamma_i}}$
$\theta_{HH} = \theta_{FF} = \theta = \frac{\alpha-1+n}{n}$

Welfare in the open economy in each country depends not only on domestic average productivity and on the number of varieties produced by local firms, but also on the total aggregate productivity $\bar{z}$ of the two economies and on the total number of varieties M produced by domestic and foreign firms.

4 Quantitative analysis

In this section I calibrate the model to match aggregate and firm level statistics of the US economy. First, I study the welfare effects of a productivity improvement in the backward country both in the closed and in the open economy. Then, I simulate the selection effect induced by trade liberalization for a given level of technology gap, and I study how a reduction in trade costs affects welfare in the two economies. I assume that in both countries the entry distribution is Pareto. The choice of this specific productivity distribution is consistent with the empirical findings on firm size distribution (e.g. Axtell (2001) and Luttmer (2007)). In this section, I relax the assumption of HRSD to the usual (first order) stochastic dominance (USD).[4] This implies that in the two countries, the productivity distributions have a common shape parameter $k_H = k_F = k$ but different scale $z_H min \geq z_F min$. Using the fact that γ is defined by the ratio of two Pareto independent random variables, I can compute $g(\gamma)$ applying formula (4) in M. Masoom Ali and Woo (2010).[5]

I calibrate nine parameters: α, τ, δ, β, λ, n, k, $z_H min$, $z_F min$. For the trade costs, I take the sum of tariff (5%) and non-tariff (8%) barriers for industrialized countries summarized by Anderson and van Wincoop (2004), and I set $\tau = 1.13$. Following Impullitti and Licandro (2010), I set $n = 6$ and $\alpha = 0.309$ getting an elasticity of substitution across varieties of 1.44, which is in the range of the estimates provided by the international business cycle literature (e.g. Heathcote and Perri (2002) and Ruhl (2008)). Impullitti and Licandro set $1/\theta_\tau = 1.13$ to match a 13% markup, which is in the range of estimation of Basu and Fernald (1994). Then, setting $n = 6$ they obtain $\alpha = 0.309$. I use the value obtained by Impullitti and Licandro also for the fixed operating costs $\lambda = 1.507$.[6]

I set $\delta = 0.09$ to match the average enterprise death rate in manufacturing in the period 1998-2004 (Census 2004). Following Rauch (1999), who finds that the differentiated goods represent a percentage between 64.4 and 67.1 of total US manufactures, we set the share of differentiated goods $1 - \beta = 0.66$. Finally, I calibrate $k = 3$ and $z_H min = 0.1$, while letting $z_F min$ vary between 0.1 and 0.01. The calibration of the shape parameter as well as of the scale parameters does not affect qualitatively our results.

Table 1 summarizes the calibration. Table 2 shows the results of the calibration in the closed economy when the foreign country is exactly half as productive

[4] Note that HRSD implies USD, but the reverse is not true.

[5] Formula (4) in M. Masoom Ali and Woo (2010) is valid for $\gamma > z_F/z_H$ .When $\gamma < z_F/z_H$ we use a transformation of γ, that is $\rho = 1/\gamma$.

[6] Impullitti and Licandro use the average firm size of 21.8 workers found in Axtell (2001) for US firms in 1997 having at least one employee.

Table 1: Summary of calibration

Parameter	Value	Moment	Source
α	0.309	Elasticity of sub/markup	Ruhl (2008)
τ	1.13	Trade cost	Anderson and van Wincoop (2004)
δ	0.09	Enterprise death rate	US Census (2004)
β	0.5	Share non differentiated	Rauch (1999)
λ	1.507	Aver.firm size	Axtell (2001)
n	6	Elasticity of sub/markup	Basu and Fernald (1994)
k	3	Std. firm productivity	Demidova (2008)
$z_{\mathrm{H}}min$	0.1	Min productivity Home	free
$z_{\mathrm{F}}min$	0.1- 0.01	Min productivity Foreign	free

Table 2: Closed economy

Parameter	Foreign	Home
$z_i min$	0.05	0.1
z_i^*	0.3418	0.6837
W_i	-1.3811	0.0054
$\bar{z}_i$	0.5127	1.0255
M_i	0.0336	0.0336
$1/\theta$	1.1250	1.1250

as the home country.

As expected, the home country has a higher productivity cutoff level and higher average productivity than the foreign country. Despite the technology gap, the home country and the foreign country produce the same number of varieties. This last finding depends on the specific form of the productivity distribution I am using, the Pareto distribution, and on the assumption of usual (first order) stochastic dominance. Consumers in the home country are better off than consumer in the foreign country, as firms in the advanced country are on average more productive (see Equation 18). Figure 3 shows a negative relation between welfare and the productivity gap in the foreign country: in the closed economy, productivity improvements in the backward country render its firms more productive and raise the welfare of its consumers.[7]

Table 3 shows the results of the calibration in open economy. In the open economy, the home country still has a higher productivity cutoff level and higher average productivity than the foreign country for any level of the productivity gap.

Productivity improvements in the backward country generate a selection effect (an increase in the productivity cutoff level and a fall in the number of varieties) in both countries (see Figure 4). However, the selection effect is stronger in the foreign country, where both the productivity cutoff level and the average productivity dramatically rise. The interpretation of this result is that when the backward country faces the productivity improvement, firms there have a better chance of receiving a high productivity draw. Therefore, firms with a low productivity which before were able to survive, exit, and the productivity cutoff rises. In the home country, instead, the selection effect is

[7] The technology gap is defined as $\gamma = \frac{Z_{Fmin}}{Z_{Hmin}}$.

Figure 3: Welfare in closed economy

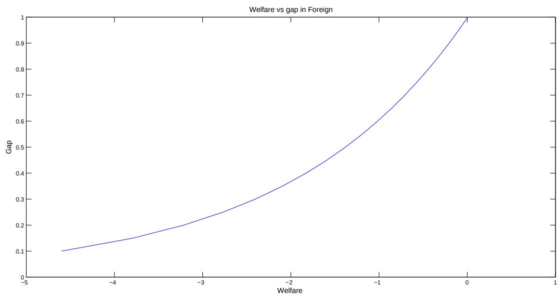


Table 3: Open economy ($z_H min = 0.1$)

$z_F min$	z_H^*	z_F^*	M_H	M_F	$\bar{z}_H$	$\bar{z}_F$	W_H	W_F
0.01	0.5769	0.0571	0.0547	0.0563	0.8653	0.0856	-0.9636	-5.4494
0.02	0.5864	0.1164	0.0522	0.0534	0.8796	0.1746	-0.9436	-4.0631
0.03	0.5971	0.1782	0.0496	0.0503	0.8957	0.2673	-0.9256	-3.2525
0.04	0.6069	0.2420	0.0473	0.0478	0.9104	0.3630	-0.9101	-2.6780
0.05	0.6149	0.3070	0.0456	0.0458	0.9224	0.4604	-0.8972	-2.2333
0.06	0.6210	0.3723	0.0443	0.0444	0.9314	0.5585	-0.8867	-1.8708
0.07	0.6252	0.4375	0.0436	0.0435	0.9378	0.6563	-0.8783	-1.5652
0.08	0.6279	0.5023	0.0430	0.0430	0.9418	0.7534	-0.8717	-1.3012
0.09	0.6293	0.5664	0.0427	0.0427	0.9439	0.8496	-0.8663	-1.0692
0.1	0.6297	0.6297	0.0426	0.0426	0.9446	0.9446	-0.8621	-0.8621

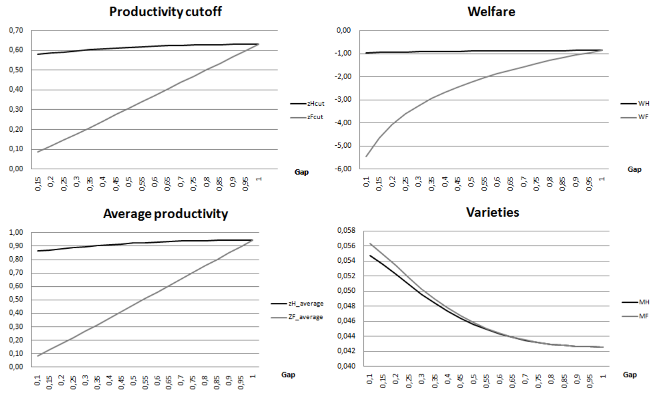
due to a more severe competition in the foreign market which forces the least productive firms to exit.

Consumers in the home country are better off than consumers in the foreign country for any level of the gap. Productivity improvements in the foreign country increase welfare in both countries, but considerably more in the backward country than in the advanced country. In both economies the positive effect on welfare is the sum of a direct effect of a reduction in the productivity gap (an increase in γ) and of an indirect effect of an increase in the average firm productivity. The sum of these two positive effects overcomes the negative effect on welfare generated by a reduction in the number of varieties (see Equations 44 and 45). Furthermore, the welfare effect is much stronger in the backward country as the average productivity there grows considerably more than the average productivity in the advanced country.

Part of my results are in line with those of Demidova (2008). She finds that the country with greater technological potential (stochastically better productivity distribution) has higher welfare per worker than the laggard country. However, she obtains partly different predictions on the welfare effects generated by productivity improvements in the backward country. Demidova shows that productivity improvements in the backward country raise the domestic productivity cutoff level there, while reducing it in the advanced country.[8] As welfare

[8]In the advanced country the export cutoff rises, whereas in the backward country falls

Figure 4: Technology catch up



in each country is an increasing function of the domestic cutoff, consumers in the laggard country gain, whereas consumers in the leading country loose.[9]

The difference in the effect that productivity improvements in the backward country generate on the productivity cutoff level and on welfare in the advanced country crucially depends on the features of the models we are using.

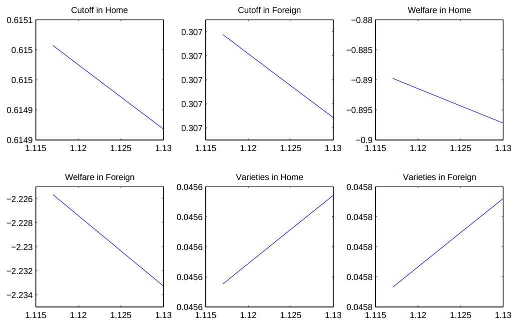
Demidova uses a Melitz (2003) framework where the domestic and the export cutoff are derived through a *free entry condition*. In her model, productivity improvements in the backward country lower the present discount value of the expected profits of firms in the advanced country. Thus, in the advanced country fewer firms enter the market and the domestic cutoff level, as well as welfare, falls. (See Demidova (2008), pp. 1454). In my model the productivity cutoff is derived through an *exit condition*. Here, productivity improvements in the backward country force the least productive firms in the advanced country to exit because of increased competition in the foreign market. As the least productive firms exit, the average productivity increases generating a positive effect on welfare. In my model, welfare is also affected directly by variations in the productivity gap (γ): in both countries, as the gap decreases (γ increases), consumers are better off.

Figure 5 shows the effects of a reduction in τ from its benchmark value of 1.13 for a given level of productivity gap ($\gamma = 0.5$). In both countries, trade liberalization generates a selection effect, thereby increasing the productivity cutoff level and lowering the number of varieties as in the baseline model of Impullitti and Licandro (2010). Finally, in both economies, a reduction in trade costs has a direct and an indirect (through increased average firm productivity) positive effect on welfare.

(See Demidova (2008) pp. 1454.

[9] Welfare per worker in Demidova (2008) is determined by the indirect utility function $W_i = (1-\beta)^{1-\beta}\beta^{\beta}\left(\frac{\beta L}{\sigma f}\right)^{\frac{\beta}{(\sigma-1)}}\left(\rho\varphi_i^*\right)^{\beta}$, where φ_i^* is the productivity cutoff for domestic producers there.

Figure 5: Trade liberalization for $\gamma = 0.5$



5 Conclusion

This paper uses a two-country model with endogenous market structure to investigate the welfare effects that productivity improvements in emerging countries generate in their developed trading partners. To my knowledge, this is the first work using an endogenous market structure framework to answer this classical question. The response of the market structure to trade liberalization (pro-competitive effect of trade) is driven by the strategic interaction of firms competing à la Cournot. Firms in the leading country draw from a stochastically better productivity distribution, thereby having a better change of receiving higher levels of productivity than firms in the laggard country. Calibrated to match firm-level and aggregate statistics of the US economy, the model predicts that the developed country has a greater productivity cutoff level, greater average productivity and greater welfare in both closed and open economy. Productivity improvements in the backward country generate more selection and positive welfare effects in both countries, with both effects being stronger in the backward country. Finally, trade liberalization, for a given level of the technology gap, leads to more selection and increases welfare everywhere. There are two general directions to extend the work presented in this paper. First, assuming differences in preferences (e.g. assuming a different elasticity of substitution across varieties in the two countries) would be more realistic in a world where countries differ in technology. Second, it would be interesting to see whether the basic results still hold in a richer environment, where only the most productive firms serve the foreign market (e.g. with fixed export costs), and where the response of the market structure to trade liberalization endogenously determines the number of firms in each industry (e.g. with an entry condition).

6 Appendix

6.1 Derivation of equation (8)

Equation (7) can be written as $x_j = \left[\tilde{z}\frac{\theta E}{X^\alpha}\right]^{\frac{1}{1-\alpha}}$. Substituting it into (2) yields

$$X^\alpha = \left(\int_0^M \tilde{z}^{\frac{1}{1-\alpha}} dj\right)^{1-\alpha} (\theta E)^\alpha$$

Combining this with the equation of x_j, I obtain

$$x = \frac{\theta E \tilde{z}^{\frac{1}{1-\alpha}}}{\left[\int_0^M \tilde{z}^{\frac{\alpha}{1-\alpha}} dj\right]^{1-\alpha}}$$

Now, substituting this into (7), using $x = nq$ and $\tilde{z} = z^{\frac{1-\alpha}{\alpha}}$, I get

$$\tilde{z}^{-1} q = \frac{(\theta E)^{1-\alpha} q^\alpha}{(M\bar{z}n)^{1-\alpha}} = \theta e \frac{z}{\bar{z}}$$

where $e = \frac{E}{(nM)}$ and $\bar{z} \equiv \frac{1}{M}\int_0^M z_j dj$

6.2 Proof of proposition 1

HRSD allows to rank expectations over an increasing function above some cutoff level, that is if $y(x)$ is increasing in x and $\Gamma_H(.) \succ_{hr} \Gamma_F(.)$, then for any given level z, $E_H[y(x) \mid x > z] > E_F[y(x) \mid x > z]$.

Using (12), I can write the EC as

$$e = \lambda \frac{1}{1-\theta}\left[\frac{1}{1-\Gamma_i(z^*)}\int_{z^*}^{\infty} z f_i(z) dz\right]\frac{1}{z^*} = \lambda \frac{1}{1-\theta} E_i\left[\frac{z}{z^*} \mid z > z^*\right], i = H, F$$

thus, since $\Gamma_H(.) \succ_{hr} \Gamma_F(.)$, given that $\frac{z}{z*}$is increasing in z and $E_i[(\frac{z}{z*} \mid z > z*] > 1, i = H, F$, it follows that

$$E_H\left[\frac{z}{z^*} \mid z > z^*\right] > E_F\left[\frac{z}{z^*} \mid z > z^*\right]$$

Therefore, for any level of z, $EC_H > EC_F$.

The proof for the MC is based on (14). Since $\Gamma_H(z^*) < \Gamma_F(z^*)$, then $M_H(z^*) > M_F(z^*)$. Consequently, for any level of z, $MC_H < MC_F$.

6.3 Firm problem in the open economy

6.3.1 First scenario: Varity j is produced only in the home (foreign) country

Let us consider the case in which variety j is produced only in the home country. Each firm there solves the following problem

$$\Pi_H = \max_{\{q_{HH}, q_{HF}\}} \left[\left(p_H - \frac{1}{\tilde{z}_H} \right) q_{HH} + \left(p_F - \frac{\tau}{\tilde{z}_H} \right) q_{HF} - \lambda \right]$$

s.t.

$$p_H = \frac{E_H}{X_H^{\alpha}} x_H^{\alpha-1}$$

$$p_F = \frac{E_F}{X_F^{\alpha}} x_F^{\alpha-1}$$

$$x_H = nq_{HH}$$

$$x_F = nq_{HF}$$

The first order conditions are

$$\left[(\alpha - 1) \frac{q_{HH}}{x_H} + 1 \right] p_H = \frac{1}{\tilde{z}_H}$$

$$\left[(\alpha - 1) \frac{q_{HF}}{x_F} + 1 \right] p_F = \frac{\tau}{\tilde{z}_H}$$

Using $x_H = nq_{HH}$ and $x_F = nq_{HF}$, multiplying the above equations by q_{HH} and q_{HF} respectively, and summing up, I obtain:

$$\frac{q_{HH} + \tau q_{HF}}{\tilde{z_H}} = q_{HH} \left[\frac{\alpha - 1 + n}{n} \right] p_H + q_{HF} \left[\frac{\alpha - 1 + n}{n} \right] p_F \tag{46}$$

Using $p_H = p_F \tau$, and $\left(\frac{x}{X}\right)^{\alpha} = \frac{z}{M\bar{z}}$, I derive the demand for variable inputs

$$\frac{q_{HH} + \tau q_{HF}}{\tilde{z}} = \psi e \frac{z_H}{\bar{z}_H} \tag{47}$$

where $\psi = \left[\frac{\alpha-1+n}{n}\right](1+\tau)$ corresponds to the inverse of the markup.

Finally, using $p_H = p_F \tau$, the first order conditions and the demand for variable inputs, I derive firms' profits

$$\pi_H \left(\frac{z_H}{\bar{z}_H} \right) = e \frac{z_H}{\bar{z}_H} \left[(1+\tau) - \psi_H \right] - \lambda$$

The specular case is when sector j is active only in the foreign country.

6.3.2 Second scenario: Varity j is active in both countries

Each firm in the home country solves the following problem

$$\Pi_H = \max_{\{q_{HH}, q_{HF}\}} \left[\left(p_H - \frac{1}{\tilde{z}_H} \right) q_{HH} + \left(p_F - \frac{\tau}{\tilde{z}_H} \right) q_{HF} - \lambda \right]$$

s.t.

$$p_H = \frac{E_H}{X_H^{\alpha}} x_H^{\alpha-1}$$

$$p_F = \frac{E_F}{X_F^{\alpha}} x_F^{\alpha-1}$$

$$x_H = n(q_{HH} + q_{FH})$$

$$x_F = n(q_{HF} + q_{HF})$$

The first order conditions are

$$\left[(\alpha - 1) \frac{q_{HH}}{x_H} + 1 \right] p_H = \frac{1}{\tilde{z}_H} \tag{48}$$

$$\left[(\alpha - 1) \frac{q_{HF}}{x_F} + 1 \right] p_F = \frac{\tau}{\tilde{z}_H} \tag{49}$$

A firm at the foreign country solves a similar problem which leads to the following first order conditions

$$\left[(\alpha - 1) \frac{q_{FF}}{x_F} + 1 \right] p_F = \frac{1}{\tilde{z}_F} \tag{50}$$

$$\left[(\alpha - 1) \frac{q_{FH}}{x_H} + 1 \right] p_H = \frac{\tau}{\tilde{z}_F} \tag{51}$$

Using (48), (49), (50), (51) and $\gamma = \frac{z_{jF}}{z_{jH}}$, I can express the markups for the domestic and the foreign market as function of the relative technology gap

$$\theta_{HH} = \frac{\alpha - 1 + 2n}{n} \left(\frac{\tilde{\gamma}}{\tilde{\gamma} + \tau} \right) \tag{52}$$

$$\theta_{HF} = \frac{\alpha - 1 + 2n}{n} \left(\frac{\tilde{\gamma}\tau}{1 + \tilde{\gamma}\tau} \right) \tag{53}$$

$$\theta_{FF} = \frac{\alpha - 1 + 2n}{n} \left(\frac{1}{1 + \tilde{\gamma}\tau} \right) \tag{54}$$

$$\theta_{FH} = \frac{\alpha - 1 + 2n}{n}\left(\frac{\tau}{\tilde{\gamma} + \tau}\right) \tag{55}$$

The market shares can be computed using the first order conditions and equations (52), (53), (54) and (55)

$$\frac{q_{HH}}{x_H} = \frac{\tilde{\gamma}(\alpha - 1 + n) - n\tau}{n(\tilde{\gamma} + \tau)(\alpha - 1)}$$

$$\frac{q_{HF}}{x_F} = \frac{\tilde{\gamma}\tau(\alpha - 1 + n) - n}{n(\tilde{\gamma} + \tau)(\alpha - 1)}$$

$$\frac{q_{FF}}{x_F} = \frac{\alpha - 1 + n - \tau\tilde{\gamma}n}{n(1 + \tau\tilde{\gamma})(\alpha - 1)}$$

$$\frac{q_{FH}}{x_H} = \frac{\tau(\alpha - 1 + n) - \tilde{\gamma}n}{n(1 + \tau\tilde{\gamma})(\alpha - 1)}$$

Using $p_F = p_H \frac{(1+\tau\tilde{\gamma})}{\tilde{\gamma}+\tau}$ and the equations of the market shares, I can derive the demand for variable inputs for a firm in the home country and for a firm in the foreign country.

Multiplying equations (48), (49), (50) and (51) by q_{HH}, q_{HF}, q_{FF} and q_{FH} respectively and summing up, I obtain

$$\frac{q_{HH} + \tau q_{HF}}{\tilde{z}_H} = \\ = \left\{ \frac{\alpha-1+2n}{n} \frac{1}{(\alpha-1)} \frac{\tilde{\gamma}}{(\tilde{\gamma}+\tau)} \left[\frac{\tilde{\gamma}(\alpha-1+n) - \tau n}{\tilde{\gamma}+\tau} + \frac{\tilde{\gamma}\tau(\alpha-1+n) - n}{1+\tau\tilde{\gamma}}\tau \right] \right\} e \frac{z}{\tilde{z}_H} \tag{56}$$

$$\frac{q_{FF} + \tau q_{FH}}{\tilde{z}_F} = \\ \left\{ \frac{\alpha-1+2n}{n} \frac{1}{(\alpha-1)} \frac{1}{(1+\tau\tilde{\gamma})} \left[\frac{\alpha-1+n-\tau\tilde{\gamma}n}{1+\tau\tilde{\gamma}} + \frac{\alpha-1+n-n\tilde{\gamma}}{\tilde{\gamma}+\tau}\tau \right] \right\} e \frac{z}{\tilde{z}_F} \tag{57}$$

Using $p_F = p_H \frac{(1+\tau\tilde{\gamma})}{\tilde{\gamma}+\tau}$, the first order conditions and the demand for variable inputs, I can now derive firms' profits in each country

$$\pi_H\left(\frac{z}{\bar{z}}\right) = \left(\frac{1}{(\alpha - 1)} \frac{1}{\tilde{\gamma} + \tau} \left[(1 + \tau)\left((\alpha - 1 + n)\tilde{\gamma} - n\right)\right] - \chi_H\right) e \frac{z}{\tilde{z}_H} - \lambda \tag{58}$$

$$\pi_F\left(\frac{z}{\bar{z}}\right) = \left(\frac{1}{(\alpha - 1)} \frac{1}{1 + \tau\tilde{\gamma}} \left[(1 + \tau)\left((\alpha - 1 + n) - \tilde{\gamma}n\right)\right] - \chi_F\right) e \frac{z}{\tilde{z}_F} - \lambda \tag{59}$$

6.4 Market clearing condition in the open economy

To derive the market clearing condition in country i, I must take into account the two possible scenarios and weigh each event by the probability that sector j is active, $1 - \Gamma_l(z_l^*)$, or not active in the other country $\Gamma_l(z_l^*)$ with $l \neq i$.

$$\left[\int_{z_H^*}^{\infty} (e\frac{z}{\bar{z}_H}\psi + \lambda)\mu_H(z)dz_H + \beta e\right]\Gamma_F(z_F*)+$$

$$\left[\int_{z_H^*}^{\infty} (e\frac{z}{\bar{z}_H}\int_{o}^{\infty} \chi_H g(\gamma)d\gamma_H + \lambda)\mu_H(z)dz_H + \beta e\right](1 - \Gamma_F(z_F*)) = \frac{1}{M(z_H^*)}$$

$$\left[\int_{z_F^*}^{\infty} (e\frac{z}{\bar{z}_F}\psi + \lambda)\mu_F(z)dz_F + \beta e\right]\Gamma_H(z_H*)+$$

$$\left[\int_{z_F^*}^{\infty} (e\frac{z}{\bar{z}_F}\int_{o}^{\infty} \chi_F g(\gamma)d\gamma_F + \lambda)\mu_F(z)dz_F + \beta e\right](1 - \Gamma_H(z_H*)) = \frac{1}{M(z_F^*)}$$

Solving for e I obtain

$$e_H = \frac{\frac{1}{M(z_H^*)} - \lambda}{\psi\Gamma_F(z_F^*) + \left[\int_o^{\infty} \chi_H g(\gamma)d\gamma\right](1 - \Gamma_F(z_F^*)) + \beta}$$

$$e_F = \frac{\frac{1}{M(z_F^*)} - \lambda}{\psi\Gamma_H(z_H^*) + \left[\int_o^{\infty} \chi_F g(\gamma)d\gamma\right](1 - \Gamma_H(z_H^*)) + \beta}$$

6.5 Welfare in the open economy

6.5.1 First scenario: Varity j is produced only in the home (foreign) country

When sector j is active only in the home country, the total quantity offered in the domestic market is $x_{jH} = n(q_{HH})$. Using $P_H = \frac{E_H}{X_H^{\alpha}} x_H^{\alpha-1}$ and (19), I can write

$$x_H = \left(\frac{X^{\alpha}}{E_H \theta_{HH} \tilde{z}_H}\right)^{\frac{1}{1-\alpha}}$$

Substituting it into $X_H = (\int_0^M x_{jH}^{\alpha} dj)^{\frac{1}{\alpha}}$ yields

$$X_H = E_H \theta (M_H \overline{z}_H)^{\frac{1-\alpha}{\alpha}}$$

where $\theta = \theta_{HH} = \frac{\alpha-1+n}{n}$.
Specularly, for the foreign country I get

$$X_F = E_F \theta (M_F \overline{z}_F)^{\frac{1-\alpha}{\alpha}}$$

where $\theta = \theta_{FF} = \frac{\alpha-1+n}{n}$.

6.5.2 Second scenario: Varity j is active in both countries

When variety j is produced in both countries, the total quantity offered in the home country is $x_{jH} = n(q_{HH} + q_{FH})$. Using $P_H = \frac{E_H}{X_H^{\alpha}} x_H^{\alpha-1}$, equations (24) and (27), and defining $\tilde{z} = \tilde{z}_H + \tilde{z}_F$, I obtain

$$x_H = \left[\frac{X^{\alpha}}{E_H} \frac{1}{\tilde{z}} \left(\frac{\theta_{FH} + \tau\theta_{HH}}{\theta_{HH}\theta_{FH}} \right) \right]^{\frac{1}{\alpha-1}}$$

Then, substituting it into $X_H = (\int_0^M x_{jH}^{\alpha} dj)^{\frac{1}{\alpha}}$ yields

$$X_H = E_H (\int_0^{\infty} \Phi_H g(\gamma) d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}$$

where $\Phi_H = [\frac{\theta_{HH}\theta_{FH}}{\theta_{FH}+\tau\theta_{HH}}] = [\frac{\alpha-1+2n}{n}] \frac{\tilde{\gamma_i}}{\tilde{\gamma_i}+\tau} \frac{1}{1+\tilde{\gamma_i}}$, M is the total number of varieties and $\bar{z}$ is total average productivity.

From the representative household problem, the homogeneous good in the home country is $Y_H = \beta E_H$.

The total quantity offered in the foreign country is $x_{jF} = n(q_{FF} + q_{HF})$. Using $P_F = \frac{E_F}{X_F^{\alpha}} x_F^{\alpha-1}$, (25) and (26) I get

$$x_F = \left[\frac{X^{\alpha}}{E_F} \frac{1}{\tilde{z}} \left(\frac{\tau\theta_{FF} + \theta_{HF}}{\theta_{HF}\theta_{FHF}} \right) \right]^{\frac{1}{\alpha-1}}$$

where $\tilde{z} = \tilde{z}_H + \tilde{z}_F$. Then, substituting it into $X_F = (\int_0^M x_{jF}^{\alpha} dj)^{\frac{1}{\alpha}}$ yields

$$X_F = E_F (\int_0^{\infty} \Phi_F g(\gamma) d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}$$

where $\Phi_F = [\frac{\theta_{FF}\theta_{HF}}{\theta_{HF}+\tau\theta_{FF}}] = [\frac{\alpha-1+2n}{n}] \frac{\tilde{\gamma_i}}{1+\tilde{\gamma_i}\tau} \frac{1}{1+\tilde{\gamma_i}}$, M is the total number of varieties and$\bar{z}$ is total average productivity.

From the representative household problem, the homogeneous good in the foreign country is $Y_H = \beta E_H$.

Finally, using (1) and taking into account the two possible scenarios, I derive welfare for consumers in the home country and for consumers in the foreign country.

$$W_H = [ln(E_H \theta (M_H \bar{z}_H)^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_H)][\Gamma_F(z^*)]+$$

$$+[ln(E_H(\int_0^\infty \Phi_H g(\gamma) d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_H)][1-\Gamma_F(z^*)]$$

$$W_F = [ln(E_F \theta (M_F \bar{z}_F)^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_F)][\Gamma_H(z^*)]+$$

$$+[ln(E_F(\int_0^\infty \Phi_F g(\gamma) d\gamma)(M\bar{z})^{\frac{1-\alpha}{\alpha}}) + \beta ln(\beta E_F)][1-\Gamma_H(z^*)]$$

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